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LETTERS TO THE EDITORS

COMMENTS ON THE PAPER "ON A DIRECT VARIATIONAL METHOD FOR NONLINEAR HEAT TRANSFER" BY B. KRAJEWSKI

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IT IS rather surprising that Krajewski does not refer to a paper that we published three years ago [1] wherein we presented a variational criterion showing a striking resemblance with the principle proposed. Our principle reads in the notation of [2]:

$$\delta Y\langle T\rangle = 0 \tag{1}$$

with

$$\begin{aligned} Y\langle T\rangle &= \int_0^t \int_V \left[\frac{\lambda^2(T)}{2}(\operatorname{grad} T)^2 + \left(\frac{\partial T}{\partial \tau}\right)^* \int_{T_0}^T C(\theta)\lambda(\theta)\,\mathrm{d}\theta\right]\mathrm{d}V\,\mathrm{d}\tau \\ &\quad + \int_0^t \int_{F_2}\left[\int_{T_0}^T \lambda(\theta)q(\theta)\,\mathrm{d}\theta\right]\mathrm{d}F\,\mathrm{d}\tau \\ &\quad + \alpha\int_0^t \int_{F_3}\left[\int_{T_0}^T \lambda(\theta)(\theta-\theta_a)\,\mathrm{d}\theta\right]\mathrm{d}F\,\mathrm{d}\tau. \end{aligned} \tag{2}$$

The upper asterisk indicates that the corresponding quantity must be kept constant during the variation. In [1] the time integral is omitted because only the time derivative appearing in the principle is fixed during the variational procedure.

Krajewski proposed for $Y\langle T\rangle$ the following expression:

$$\begin{aligned} Y\langle T\rangle = &\int_0^t \int_V \left[\frac{\lambda^2(T)}{2}(\operatorname{grad} T)^2 + F\int_{T_0}^T C(\theta)\lambda(\theta)\,\mathrm{d}\theta - \int_{T_0}^T W(\theta)\lambda(\theta)\,\mathrm{d}\theta\right]\mathrm{d}V\,\mathrm{d}t \\ &+ \int_0^t \int_{F_2}\left[\int_{T_0}^T q(\theta)\lambda(\theta)\,\mathrm{d}\theta\right]\mathrm{d}F\,\mathrm{d}\tau \\ &+ \alpha\int_0^t \int_{F_3}\left[\int_{T_0}^T (\theta^m-\theta_a^m)\lambda(\theta)\,\mathrm{d}\theta\right]\mathrm{d}F\,\mathrm{d}\tau \\ &+ Z|_{t=0} = \text{minimum} \end{aligned} \tag{2'}$$

where

$$F = \frac{\partial T}{\partial \tau}$$

is supposed to be a given function of x, y, z and τ while $Z|_{t=0}$ is defined by

$$Z|_{t=0} = \tfrac{1}{2}\textstyle\int_V (T-T_p)^2\,\mathrm{d}V. \tag{3}$$

T_p is the initial temperature distribution.

The differences in expressions (2) and (2′) lay in the introduction by Krajewski of a heat source term, a more general radiation law and a $Z|_{t=0}$ term. It is a rather trivial extension to incorporate in (2) heat sources and general radiation boundary conditions. The Z term has no direct link with the expression (2′) of the principle because it may be added independently to the expression of $Y\langle T\rangle$; this is confirmed by the applications considered by Krajewski wherein the least square principle

$$\delta\textstyle\int_V (T-T_p)^2|_{t=0}\,\mathrm{d}V = 0$$

is clearly used independently of the criterion (2′) itself.

A crucial point in Krajewski's presentation is that $\partial T/\partial\tau$ must be a known function of the space coordinates and the time. But if this were known, a simple time integration would directly yield the temperature distribution. No variational principle is needed then. Actually, as it emerges clearly from the numerical analysis, Krajewski does not consider $\partial T/\partial\tau$ as given *a priori* but he keeps this term constant during variation. Otherwise, it would not be possible to derive relations such as (34) and (41).

For this reason, the principle proposed by Krajewski must clearly be classified as a restricted one and as shown by Finlayson and Scriven [3], the functional (2′) is not a minimum. Therefore the claim made by the author that "the formulation described in the present paper satisfy the conditions necessary for the existence of an extremum of variational integral" is not correct.

Moreover, even if $\partial T/\partial\tau$ would be a known function of time and position, the sign of the second variation $\delta^2 Y\langle T\rangle$ would generally not be positive definite; therefore, (2′) cannot be referred as a minimum principle.

Finally, we must also point out that contrary to Krajewski's assertion, there is no reason to believe that Kantorovitch's method is more accurate than Rayleigh–Ritz's. The crucial point is the choice of the trial function. Whatever the method selected, the quality of the results is essentially determined by the nature of the trial function and also, of course, by the accuracy of the numerical techniques involved. In most cases, Rayleigh–Ritz's method is recommended because it leads to a set of algebraic equations instead of differential equations.

Department of Mechanics
University of Liège
15 Avenue des Tilleuls
4000 Liege
Belgium

G. LEBON
J. LAMBERMONT

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